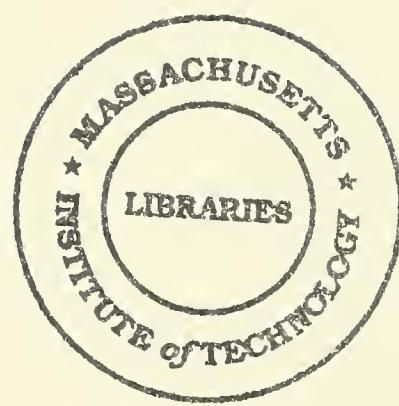


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**THE NOISY DUOPOLIST**

David Spector

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December 2000

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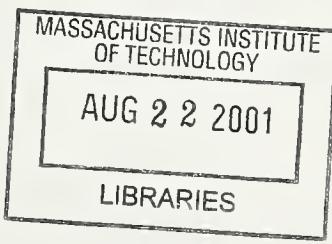
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# The noisy duopolist\*

David Spector (MIT)<sup>†</sup>

December, 2000 (first version: June, 2000)

## Abstract

This paper provides an explanation for noisy pricing based on the strategic interaction of two firms competing in prices. When a firm adds noise to its prices, undercutting it becomes harder. Therefore, noisy pricing allows a firm to either exclude a competitor while charging supracompetitive prices, or to soften competition and have both firms earn supracompetitive profits. Such behavior leads to prices lying between the competitive and monopolistic levels, and harms consumers and social welfare. It occurs in equilibrium if firms set prices sequentially, and in some equilibria of a repeated game of simultaneous price-setting if one firm is patient.

## 1 Introduction

Firms sometimes seem to introduce noise into their pricing behavior. This noise can take the form of unadvertised sales, vagueness about product specifications, or price differences across branches. Telephone companies make specific offers to randomly selected individuals by telephone, airline fares or insurance premia are notoriously difficult to understand, supermarkets offer coupons with a random discount, and so on. Several explanations have been put forward in the economic literature. This paper proposes a new and extremely simple one.

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Most of the literature has argued that such practises allow a monopolist to exploit heterogeneity among consumers. For example, price discrimination allows to extract more from high-valuation / low elasticity consumers without losing other consumers, who are offered a lower price; price dispersion similarly allows to exploit heterogeneity in search costs.<sup>1</sup>

This paper shows that price dispersion may be profitable for another reason, even if consumers are identical. The argument is the following: by making its prices noisy, a firm makes it harder for a competitor to undercut its price. This may allow a firm to deter entry or induce exit of an equally efficient competitor while earning strictly positive profits, or, without necessarily excluding a competitor, to soften competitive pressures<sup>2</sup>.

The argument relies on the assumption that when a firm (say, firm 1) introduces noise into its prices, by offering different prices to observationally identical consumers, the competitor or potential entrant (say, firm 2) cannot observe which price a given consumer was offered, but only the distribution of prices offered by firm 1. If on the contrary firm 2 could observe the price offered by firm 1 to each consumer, the usual logic of contestable markets would prevail: the threat of being undercut by firm 2 would suffice to bring firm 1's prices down to the level leading to zero profits.

However, the undercutting argument breaks down if individual price offers cannot be observed. When offering a price to a given consumer, firm 2 must take into account the trade-off between a high profit margin and the probability that the consumer, being offered a lower price by firm 1, will reject the offer. Therefore, the nature of the strategic interaction between firms dramatically changes when firms compete in price distributions rather than in prices.

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<sup>1</sup>The topic of price discrimination has been addressed by many papers. For surveys, see Phlips [1983], Varian [1989] and Chapter 3 of Tirole [1990]. The idea that "noisy pricing" allows to exploit consumer heterogeneity appears under various forms in Salop [1977], Salop and Stiglitz [1977], Pratt, Wise and Zeckhauser [1979], and Varian [1980].

<sup>2</sup>A few papers describe how price dispersion may arise in a world with identical consumers. Salop and Stiglitz [1982] model price dispersion in a competitive world (unlike this paper, which focuses on the strategic interaction between two firms). Wilson [1998] shows that a monopolist facing quantity constraints may find it optimal to price some units of its good higher than others, but the result is relatively weak (the monopolist never charges more than two different prices) and requires strong assumptions. Price dispersion may also be used as a way to deter the formation of consumer coalitions which would integrate into production (Innes and Sexton [1993]). Butters [1977], probably the closest paper to this one, models price dispersion within a model of costly advertising.

Even in a perfectly contestable market - in the sense that there are no sunk costs, firms have access to the same technology, and a firm cannot quickly change its prices in response to entry - prices exceed average cost in equilibrium, and there is price dispersion in equilibrium. In some cases, a firm finds it profitable to make its prices noisy in order to deter entry, or to induce a competitor's exit. In other cases, noisy pricing allows to accommodate entry, i.e. to earn positive profits in spite of the presence of a competitor in the same market.

For example, if technologies exhibit constant returns to scale and firm 1 offers some consumers a price above marginal cost, then it is impossible to prevent firm 2's entry or to induce its exit: firm 2 finds it profitable indeed to be present in the market and to offer all consumers a price slightly below firm 1's highest price, because it will be accepted by some consumers, and lead to positive profits. But even though noisy pricing cannot lead to firm 2's exclusion, firm 1 finds it attractive: by carefully choosing the distribution of its price offers, it can make it optimal for firm 2 to charge a high price and be selected by a small fraction of the consumers, rather than earning low margins from a larger population. This leaves some consumers to firm 1, and this strategy leads to a division of the market between both firms, with both earning positive profits.

These results may contribute to our understanding of markets on several grounds. First, they provide a new rationale for price dispersion which, unlike most existing explanations, does not rely on any heterogeneity among consumers. Second, the equilibrium can be interpreted as the Stackelberg equilibrium of a price-distribution setting game where both firms move simultaneously, rather than in terms of a sequential model: the incumbent and the (potential) entrant can be seen, respectively, as the Stackelberg leader and follower. Equivalently, it can be seen as an equilibrium of a game of repeated interaction between firms, where only one firm is patient - while collusive outcomes and monopolistic prices require both firms to be patient. Since the outcome highlighted in this paper lies somewhere between pure competition and full collusion (in terms of prices and profits), this model may help to think about price competition in terms that are more nuanced than those allowed by the usual opposition between Bertrand competition and collusion.

The paper is organized as follows: the outcome of competition in prices and competition in price distributions with sequential moves are compared under an extreme form of increasing returns (section 2) and under constant

returns (section 3). In Section 4, it is shown that the outcomes highlighted in sections 2 and 3 can be interpreted as equilibria of a repeated game where firms simultaneously choose price distributions, provided that one of the firms is patient enough. Section 5 concludes.

## 2 The case of increasing returns

### 2.1 Technology and preferences

There is a continuum of agents of mass one, who derive utility from the consumption of some good and from money. They have identical preferences, characterized by the following utility functions, where  $x$  denotes the consumption of the good and  $m$  denotes the amount of money:

$$\begin{cases} U(x, m) = m & \text{if } x < 1 \\ U(x, m) = m + A & \text{if } x \geq 1. \end{cases} \quad (1)$$

In other words, consumers have a unit demand for the good, and a valuation of  $A$ . Two firms, an incumbent (firm 1) and a potential entrant (firm 2) can produce the good, using the same technology, which is characterized by an extreme form of increasing returns. The cost function  $c_1$  is indeed given by

$$\begin{aligned} c_1(Y) &= 0 & \text{if } Y = 0 \\ c_1(Y) &= B & \text{if } Y > 0. \end{aligned}$$

*Assumption 1:*  $A > B$ .

Assumption 1 implies that it is efficient for the good to be produced. Firms are assumed to maximize profits, and consumers maximize utility.

### 2.2 The benchmark: competition in prices

This section considers the benchmark case of competition in prices, described by the following very simple model:

- Step 1. Firm 1 sets a price  $p_1$ .
- Step 2. Firm 2 sets a price  $p_2$ .

Step 3. Every consumer chooses whether to buy the good or not, and from whom.

Step 4. Production, exchange and consumption takes place.

This market structure corresponds to a contestable market in the sense of Baumol et al. [1982]: there are no sunk costs, both the incumbent and the potential entrant have the same technology, and the incumbent cannot respond to entry by lowering its price. In this case, it is well-known that, although in equilibrium no entry occurs, the threat of entry is enough to drive the incumbent's price down to the point where profits are zero. This is summarized in

**Proposition 1** *The only equilibria of this game are such that the incumbent sets  $p_1 = B$ , and the potential entrant chooses not to enter (i.e., it sets  $p_2 > B$ ). Every consumer consumes one unit of the good and the incumbent earns zero profits.*

### 2.3 Competition in price distributions

Let us consider now the following modification. Instead of setting a price, each firm sets a probability distribution. More precisely, each firm can offer different prices to different consumers, and the potential entrant can only observe the distribution of the incumbent's prices, but not the actual price offered to specific individuals.

This assumption does not correspond to a world where firms discriminate conditionally on observable individual characteristics, but rather to one where the price offered to a given consumer is random, driven from some distribution. This may capture behavior such as random or unadvertised sales, or special offers made by telephone, by mail, or on the internet. The important assumption is that a consumer may take a firm's price as given and compare it to the price offered by a rival (so that firms can commit), although it cannot show the rival evidence about the price it was offered by the other firm. Formally, this market structure is modeled by the following game:

Step 1. Firm 1 chooses a price distribution  $\mu_1$ .

Step 2. Firm 2 observes  $\mu_1$  and chooses a price distribution  $\mu_2$ .

Step 3. Every consumer  $i$  is offered a price  $p_{i1}$  by firm 1, randomly drawn from the distribution  $\mu_1$ , and a price  $p_{i2}$  by firm 2, randomly drawn from the distribution  $\mu_2$ . The draws of  $p_{i1}$  and  $p_{i2}$  are independent from each other and across consumers.

Step 4. Every consumer chooses whether to buy the good or not, and from which firm.

Step 5. Production, exchange and consumption takes place.

If firm 2 chooses a price distribution generating zero sales, this will be interpreted as a choice not to enter into the market. Obviously, firm 2 can choose not to enter simply by offering prices above  $B$  to all consumers. This game leads to a very different equilibrium:

**Proposition 2** Consider the unique solution  $\eta$  to the equation  $x = e(\text{Log}(x) + 1)$

- **Case 1:**  $\frac{A}{B} < \eta$ . If  $\frac{A}{B} < \eta$ , then in equilibrium the incumbent offers prices drawn from the probability distribution  $\mu^*$  defined by

$$\begin{cases} \mu^*([0, B]) = 0; \\ \text{If } B < p < A, \text{ then } d\mu^*(p) = \frac{B}{p^2}; \\ \mu^*(\{A\}) = \frac{B}{A}; \\ \mu^*((A, \infty)) = 0. \end{cases}$$

The potential entrant chooses not to enter. Each consumer buys one unit of the good, at a price equal, in average, to  $B(1 + \text{Log}(\frac{A}{B}))$ . The incumbent's profit is equal to  $B\text{Log}(\frac{A}{B})$ .

- **Case 2:**  $\frac{A}{B} > \eta$ . If  $\frac{A}{B} > \eta$ , then in equilibrium, the incumbent offers prices drawn from the probability distribution  $\hat{\mu}$  defined by

$$\begin{cases} \hat{\mu}([0, \frac{A}{e}]) = 0; \\ \text{If } \frac{A}{e} < p < A, \text{ then } d\hat{\mu}(p) = \frac{A}{ep^2}; \\ \hat{\mu}(\{A\}) = \frac{1}{e}; \\ \hat{\mu}((A, \infty)) = 0. \end{cases}$$

The potential entrant then offers to sell the good at price  $A$ , and almost all consumers offered the price  $A$  by both firms buy from the entrant. Each consumer consumes one unit of the good, and each firm makes profits equal to  $\frac{A}{e} - B$ .

**Proof.** See the appendix. ■

*Remarks.* 1. The argument can be summarized as follows. In order to deter entry, the incumbent's price distribution must be such that the expected revenue that the entrant would earn when offering *any* price be at most equal to the fixed cost  $B$ . Such a price distribution may have an average greater than  $B$ , because the uncertainty about the price a given consumer was offered by the incumbent implies that if the entrant offers a price greater than  $B$ , it is rejected with a positive probability. However, deterring entry requires that the incumbent's distribution put enough weight on low prices. For example, if  $B = 0$ , the maximal profit compatible with entry deterrence is zero. If the production cost is small enough relative to consumers' valuations (more precisely, if  $\frac{A}{B}$  is greater than  $\eta$ ), the incumbent prefers not to deter entry, but rather to induce entry at high prices, which allows to retain part of the market. This can be achieved by choosing a price distribution such that the entrant's trade-off between high prices and low probabilities of acceptance is solved in favor of high prices: the market is then divided between the entrant, which sells to a few consumers at high prices, and the incumbent, which sells to more consumers, but at lower prices.

2. One way to characterize the result is to say that noisy pricing can be used either to deter or to accommodate entry (whichever is more profitable, which depends on whether  $\frac{A}{B}$  is greater or smaller than  $\eta$ ). It is quite striking that the shape of the incumbent's price distribution is very similar in both cases. In the case where accommodation is optimal, the result can be explained as follows: in order to alleviate competition after entry takes place, the incumbent wants the entrant's residual demand function not to be too elastic, in order to induce it to charge high prices. In the real world, product differentiation implies a firm's residual demand (given another firm's price) to have a finite elasticity, which leads it to charge high enough prices, and allows both firms to earn positive profits, both in the Nash equilibrium of the price-setting game and in the Stackelberg equilibrium. When products are identical, the only way for the incumbent to reduce the elasticity of the entrant's residual demand, and therefore to allow for positive profits, is to make its own prices noisy.

3. When noisy pricing is used to accommodate entry, entry occurs although it is socially inefficient: the presence of two firms is wasteful because it duplicates the fixed cost. If the demand function is smoother than the one assumed above, the possibility of price discrimination is also socially wasteful

because the higher prices it induces cause demand to decrease and output to be at a suboptimal level.

4. The average price is somewhere between the zero-profit price ( $B$ ), which would prevail if price discrimination were impossible, and the monopoly price ( $A$ ). In particular, equilibrium prices are greater for all consumers than in the case of no discrimination. This contrasts sharply with the results of Salop [1977] Schmalensee [1981] and Varian [1985], who find that price discrimination (or price dispersion) leads, for some buyers, to lower prices than uniform pricing<sup>3</sup>.

### 3 The case of constant returns

This section shows that the assumption of increasing returns does not drive the results. I assume that the technology displays constant returns to scale, and show that the incumbent can still make strictly positive profits, in spite of entry by the competitor. Therefore, even when there is no minimal profitable scale of production, the inobservability of individual prices is enough to weaken competition and allows both firms to earn strictly positive profits.

#### Technology

The only difference between this section and the previous one is that the technology available to both firms is now characterized by the cost function  $c_2$  given by

$$c_2(Y) = BY.$$

Assumption 1 is still made.

#### 3.1 The one-shot game

**Proposition 3** *Assume that both firms' cost function is  $c_2$  (linear) and that firms sequentially compete in price distributions - i.e., they play the same*

---

<sup>3</sup>Katz [1987] showed that this result cannot be extended to the case of intermediate good markets: discrimination may cause prices to rise for all buyers, as in our model, but for reasons completely different from the ones emphasized in this paper.

game as in Section 2.3). Then the only equilibrium of this game is the following: the incumbent offers prices drawn from the distribution  $\tilde{\mu}$  defined by

$$\begin{cases} \tilde{\mu}([0, B + \frac{A-B}{e}]) = 0; \\ \text{If } B + \frac{A-B}{e} < p < A, \text{ then } d\tilde{\mu}(p) = \frac{A-B}{e(p-B)^2}; \\ \tilde{\mu}(\{A\}) = \frac{1}{e}; \\ \tilde{\mu}((A, \infty)) = 0. \end{cases}$$

The potential entrant then offers to sell the good at price  $A$ , and almost all consumers offered the price  $A$  by both firms buy from the entrant. Each consumer consumes one unit of the good, and the profits of the incumbent firm and of the entrant are both equal to  $\frac{A-B}{e}$ .

**Proof.** Consider an equilibrium of the game. Clearly, the choice of a price  $p$  can be reformulated as the choice of the difference  $p' = p - B$ , and the game with a constant marginal cost of  $B$  and a valuation of  $A$  for consumers is equivalent to one where the marginal cost is zero and consumers' valuation is  $A - B$ . But the case of a zero marginal cost can be seen as a particular case of Proposition 2, with  $B = 0$ . Applying Case 2 of Proposition 2 with  $B = 0$  implies that in equilibrium there is entry, and that the distribution of  $p' = p - B$  induced by the incumbent's strategy is given by

$$\begin{cases} \mu([0, \frac{A-B}{e}]) = 0; \\ \text{If } \frac{A-B}{e} < p < A - B, \text{ then } d\mu(p) = \frac{A-B}{ep^2}; \\ \mu(\{A - B\}) = \frac{1}{e}; \\ \mu((A - B, \infty)) = 0, \end{cases}$$

implying that the distribution of prices is  $\tilde{\mu}$ . ■

### Interpretation in terms of Stackelberg competition

All the results stated so far can be formulated in terms of Stackelberg equilibria rather than in terms of a sequential game: the only important element is that one of the firms takes into account the effects of its pricing decision on the other firm's pricing decision. This interpretation may be more appealing than the one in terms of entry for industries where several competitors are already there.

The following proposition clarifies this point, by showing that the Nash equilibrium of the pricing game is the same whether firms compete in price

distributions or in prices, as in the usual Bertrand competition model. This implies that the sequential nature of the game (or equivalently the assumptions of a Stackelberg equilibrium rather than a Nash equilibrium) is crucial for the results<sup>4</sup>.

**Proposition 4** *Assume that firms simultaneously choose their price distributions. Then, if the technology is described by the cost function  $c_2$ , the only Nash equilibrium is such that both firms offer to sell the good at price  $B$  with probability one and make zero profits.*

**Proof.** See the Appendix. ■

By continuity, entry also occurs in the sequential game of competition in price distributions when returns to scale are mildly increasing: if the cost function is  $c(Y) = BY^\alpha$ , with  $\alpha \in [0, 1]$ , the cost functions  $c_1$  and  $c_2$  correspond to  $\alpha = 0$  and  $\alpha = 1$  respectively, and Proposition 3 and a continuity argument imply that entry takes place for values of  $\alpha$  close to 1. But as soon as  $\alpha$  is strictly smaller than one, increasing returns imply that entry is socially wasteful: random price discrimination by the incumbent / Stackelberg leader triggers inefficient entry. This is summarized in the following table:

|                                    | Nash                     | Stackelberg   |
|------------------------------------|--------------------------|---|
| competition in prices              | Zero profits<br>No entry | Zero profits<br>No entry                                |
| competition in price distributions | Zero profits<br>No entry | Positive profits<br>Too many firms<br>Inefficient entry |

---

<sup>4</sup>Baye and Morgan [1999] show that a simultaneous price-setting game may be characterized by Nash equilibria where firms randomize prices and earn strictly positive profits, in addition to the standard zero-profit equilibrium. But this possibility requires the existence of a strictly positive demand for any price, however high, which seems very implausible. Under fairly weak assumptions, such as those made in this paper, the zero profit outcome is the only equilibrium (Baye and Morgan [2001]).

## 4 The dynamic game: between the Bertrand outcome and collusion

The outcome of the Stackelberg equilibrium of the game in price competitions described above is intermediate between the Nash-Bertrand equilibrium and the outcome of collusion between both firms, because total profits, equal to  $\frac{2(A-B)}{e}$ , are greater than the Nash-Bertrand zero profits, but smaller than the monopolistic profit level  $A - B$  achieved in the case of collusion.

Since collusion is usually analyzed in dynamic settings, a repeated game can clarify this comparison. I assume that the above game is repeated infinitely: in every period, there is a mass one of consumers, and both firms (labeled firm 1 and firm 2) simultaneously offer a price to every consumer, possibly different across consumers. Both firms' cost functions are  $c_2(Y) = BY$ , and all consumers' preferences are still characterized by (1), with  $B < A$ . Consumers live for one period (a new generation appears every period), and firm  $i$  maximizes in period  $t$  its flow of future discounted profits  $\sum_{t' \geq t} \beta_i^{t'} \pi_i(t')$ , where  $\pi_i(t')$  is firm  $i$ 's profit in period  $t'$  and  $\beta_i < 1$  is firm  $i$ 's rate of time preference.

As always, the repetition of the static Nash equilibrium is an equilibrium of the dynamic game. Proposition 4 implies that this equilibrium has both firms offering a price  $B$  to all consumers, and making zero profits. The standard results of the theory of collusion (see, e.g., Tirole [1988], pp. 245-246) also tell us that if  $\beta_1 > \frac{1}{2}$  and  $\beta_2 > \frac{1}{2}$ , then the collusive outcome, where firms jointly behave as a monopoly, offering the price  $A$  and earning, each, a profit of  $\frac{A-B}{2}$ , is another equilibrium of the repeated game. The following proposition shows that the conditions needed to have price dispersion and positive profits in some equilibria of the dynamic game are weaker than the ones needed to make collusion possible.

**Proposition 5** *If  $\beta_2 = 0$  and  $\beta_1 \geq \frac{e-1}{e}$ , then there exists an equilibrium of the repeated game such that:*

- (i) *Each of firm 1's offers is drawn from the distribution  $\tilde{\mu}$  as defined in Proposition 3.*
- (ii) *Firm 2 offers the price  $A$  to all consumers.*
- (iii) *Each firm earns profits equal to  $\frac{A-B}{e}$ .*

**Proof.** First, offering  $B$  to all consumers is a best response for firm 2 if firm 1's distribution of price offers is  $\tilde{\mu}$ . I assume, as in the standard theory

of collusion, that if any firm deviates from the strategies mentioned in the proposition, then the equilibrium of the continuation game is the repeated static equilibrium. This implies that, if firm 1 deviates from the distribution  $\tilde{\mu}$  in the current period, all future profits are zero for both firms. Firm 1's maximum gain from deviating is obtained by offering a price slightly below  $A$  and serving all the consumers: this would cause profit to rise from  $\frac{A-B}{e}$  to  $A - B$ .  $\tilde{\mu}$  is optimal, therefore, if  $\frac{A-B}{e} \sum_{t \geq 0} \beta_1^t \geq A - B$ , or  $\beta_1 \geq \frac{e-1}{e}$ . ■

This result lends itself to the following interpretation. The equilibrium with price dispersion and positive profits, which is intermediate (in terms of profits) between the Nash-Bertrand outcome and the collusive outcome, arises under intermediate assumptions. If both firms are very impatient, then the Nash-Bertrand outcome is the only equilibrium of the repeated game. If both are patient enough, then collusion is an equilibrium. But if only one of them is patient, then the equilibrium with price dispersion is an equilibrium.

This result may provide a way for the analysis of price competition to depart from the usual polarization between the extreme cases of the zero-profit Bertrand outcome and full collusion, which are both considered implausible in many settings. If firms compete in price distributions rather than in prices, the assumption that only *one* firm is sufficiently patient to behave strategically is enough to yield prices strictly above marginal costs for *both* firms, while still being lower than collusive prices<sup>5</sup>.

## 5 Conclusion

The conclusions derived from the model developed in this paper are of course sensitive to the very specific assumptions. In particular, it may be hard to extend the model to more than two firms: if firms simultaneously choose price distributions, the presence of two impatient firms is enough to bring all firms' profits to zero. More generally, the relative magnitude of the effect stressed here and the well-known phenomenon of discrimination among heterogeneous consumers is a question of paramount importance to assess the relevance of this paper, and should be the subject of future research. At the very

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<sup>5</sup>The condition presented here is weaker than the one required for collusion to be an equilibrium outcome in the sense that only one firm needs to be patient. But, strictly speaking, it is not a weaker condition, because the degree of patience required of the only patient firm is greater than that required of both firms for collusion to arise (the minimum value for  $\beta_1$  is  $\frac{e-1}{e}$ , which is greater than  $1/2$ ).

least, many of the real-world examples of price dispersion mentioned in the literature seem consistent with the theory developed in this paper.

The model led to three main findings. First, noisy pricing may occur in the absence of any heterogeneity among consumers, as a strategic tool used by an incumbent firm (or a Stackelberg leader, or a patient firm) to either exclude a rival while charging supracompetitive prices, or influence the pricing decisions of an entrant (or a Stackelberg follower, or a less patient firm) in order to soften competition and allow both firms to earn strictly positive profits. Second, it adversely affects all consumers and is socially harmful. Third, when noisy pricing is used to accommodate entry, it does not harm the entrant (contrary, for example, to Armstrong and Vickers [1993], where price discrimination is used by an incumbent to deter entry), but it rather benefits both firms at the expense of consumers. Interestingly, noisy pricing is most harmful to welfare precisely when it benefits the entrant (or Stackelberg follower, or impatient firm).

This last point may have some policy implications. Assume that, wary of the effects described in this paper, the government enacts regulations to limit noisy pricing. When a firm makes its prices noisy for the reasons outlined in this paper, adversely affecting consumers, the competition authority cannot count on potential or actual competitors to report this behavior and have the regulations enforced, because these competitors are likely to benefit from this price dispersion. Therefore, if this model has any validity for the real world, it suggests that governments should actively promote transparent information about prices. Such a policy, however, would be the exact opposite of what a government concerned with the possibility of collusion should do, because price transparency facilitates collusion. The contrast between the policy prescriptions associated with these two views of the world implies that assessing the empirical relevance of the mechanism outlined in this paper might be very useful.

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## APPENDIX

### PROOF OF PROPOSITION 2

**PART I.** Let us start by considering a hypothetical equilibrium such that the potential entrant decides not to enter, and let  $\mu_1$  denote the incumbent's strategy in this equilibrium.

**Result 1.**  $\mu_1$  solves the following optimization problem:

$\text{Max}_p E(p \cdot 1_{p \leq A} | \mu)$  under the constraint

$$\text{Max}_p p \mu([p, \infty)) \leq B.$$

*Proof of Result 1.* An integration by parts between 0 and  $A$  implies that if the potential entrant does not enter and the incumbent plays a strategy  $\mu$ , the incumbent's profit is equal to

$$E(p \cdot 1_{p \leq A} | \mu) = \int_0^A \mu([p, A]) dp.$$

Writing  $G_\mu(p) = \mu([p, \infty))$ , it implies that

$$E(p \cdot 1_{p \leq A} | \mu) \leq \int_0^A G_\mu(p) dp.$$

But the problem

$$\begin{aligned} & \text{Max} \int_0^A G(p) dp \text{ under the constraint} \\ & G(p) \leq \text{Min}(1, \frac{B}{p}) \end{aligned}$$

has a unique solution  $G^*$  given by

$$G^*(p) = \text{Min}(1, \frac{B}{p}) \text{ for } p \leq A,$$

and if  $\mu$  is defined by  $\mu([p, A]) = \text{Min}(1, \frac{B}{p})$ , we have

$$E(p \cdot 1_{p \leq A} | \mu) = \int_0^A G^*(p) dp,$$

and for every probability distribution  $\mu$  such that  $\max_p \mu([p, \infty)) \leq B$ ,

$$E(p \cdot 1_{p \leq A} | \mu) \leq \int_0^A G_\mu(p) dp \leq \int_0^A G^*(p) dp = E(p \cdot 1_{p \leq A} | \mu_1),$$

which proves Result 1.

**Result 2.** Let us modify the game by assuming that entry inflicts the incumbent an infinite utility loss. Then the only subgame-perfect equilibrium of the modified game is the one described in the first part of the proposition, irrespective of the value of  $\frac{A}{B}$ .

*Proof of Result 2.* Let us consider the potential entrant's entry decision. In any subgame perfect equilibrium, it decides to enter (resp. not to enter) with probability 1 if the incumbent's price distribution allows it to make a strictly positive (resp. strictly negative) profit. Assume the distribution of the incumbent's price offers is  $\mu$ . The potential entrant's expected revenue from a randomly chosen consumer to whom it offers  $p$  is  $p\mu([p, \infty))$ , so the potential entrant's profit if it decides to enter and sets a profit-maximizing price distribution has the same sign as

$$\pi(\mu) = \max_p p\mu([p, \infty)) - B.$$

Let us prove first that the strategies described in the proposition are an equilibrium pair of strategies. It is indeed an equilibrium strategy for the potential entrant to enter only if  $\pi(\mu) > 0$ . If the potential entrant follows this strategy, then consider a hypothetical strategy  $\mu$  such that  $\pi(\mu) > 0$ . If chosen by the incumbent, this strategy triggers entry, and the potential entrant's revenue exceeds  $B$ . Therefore the incumbent's revenue is less than  $A - B$  (the sum of both firms' revenues is indeed less than  $A$ ), and its profit is less than  $A - 2B < 0$ . On the contrary, the choice of  $\mu^*$  does not cause entry (because  $\pi(\mu^*) = 0$ ) and yields a positive profit (all prices are above  $B$ ). This implies that the optimal strategy for the incumbent should not trigger entry. Since the revenue generated by a distribution  $\mu$  is  $E(p \cdot 1_{p \leq A} | \mu)$ , this implies that the incumbent's optimal strategy maximizes  $E(p \cdot 1_{p \leq A} | \mu)$  under the constraint  $\pi(\mu) \leq 0$ . Therefore, the incumbent's best strategy in any equilibrium where the entrant only enters when it can earn a strictly positive profit is  $\mu^*$ . Let  $\Pi$  denote  $E(p \cdot 1_{p \leq A} | \mu^*) = E(p | \mu^*)$ .

We prove now that there are no other equilibria, that is, that there are no subgame-perfect equilibria where the entrant enters with a positive prob-

ability when it can earn zero profits. Assume that there exists such an equilibrium. First, the same argument as in the above paragraph implies that any  $\mu$  such that  $\pi(\mu) > 0$  is dominated, from the incumbent's viewpoint, by some strategy  $\mu'$  close enough to  $\mu^*$ , such that  $\pi(\mu') < 0$ . This argument can be applied to show that in equilibrium, the incumbent's strategy is  $\mu^*$ . Let  $\mu^0$  denote the incumbent's strategy, and let us assume that  $\mu^0 \neq \mu^*$ . It was just shown that  $\pi(\mu^0) \leq 0$ , and this implies that the incumbent's expected profit is strictly less than  $\Pi$ . But consider, for  $\varepsilon$  small enough, the strategy  $\mu(\varepsilon)$  maximizing  $E(p \cdot 1_{p \leq A} | \mu)$  under the constraint  $\pi(\mu) \leq -\varepsilon$ . Clearly,

$$\lim_{\varepsilon \rightarrow 0} E(p \cdot 1_{p \leq A} | \mu(\varepsilon)) = \Pi,$$

and  $\mu(\varepsilon)$  does not trigger entry if  $\varepsilon > 0$ . This implies that  $\mu(\varepsilon)$  dominates  $\mu^0$  if  $\varepsilon$  is small enough, leading to a contradiction.

**PART 2.** Assume that there exists an equilibrium such that the potential entrant enters. Then this equilibrium is the one described in the second part (Case 2) of the Proposition.

**Result 1.** If  $\mu$  is an equilibrium strategy for the incumbent, in an equilibrium where there is entry with a positive probability, then

$$A \in \operatorname{Arg} \max_p \mu([p, \infty)).$$

*Proof of Result 1.* Assume that the result is not true, so that  $\operatorname{Arg} \max_p \mu([p, \infty)) = A' < A$ . This implies that for any  $\varepsilon > 0$ ,  $\mu([A', A' + \varepsilon]) > 0$ . Let us choose some positive  $\varepsilon$  and consider the price distribution  $\mu'$  given by

$$\begin{aligned} \mu'(S) &= \mu(S) \text{ if } [A', A' + \varepsilon] \not\subseteq S \\ \mu'([A', A' + \varepsilon]) &= 0 \\ \mu'(\{A' + \varepsilon\}) &= \mu([A', A' + \varepsilon]). \end{aligned}$$

Clearly,  $\operatorname{Arg} \max_p \mu'([p, \infty)) \subset [A', \infty)$ . If we define  $\mu_\lambda = \lambda \mu' + (1 - \lambda) \mu$ , it is still true that for every  $\lambda \in (0, 1)$ ,  $\inf \left( \operatorname{Arg} \max_p \mu_\lambda([p, \infty)) \right) > A'$ . If the incumbent plays  $\mu$ , then the entrant sets a price smaller or equal than  $A'$ , while the last inequality ensures that the entrant sets a price strictly greater than  $A'$  if the incumbent plays  $\mu_\lambda$  with  $0 < \lambda < 1$ . The fact that  $\mu_\lambda([A', A' + \varepsilon]) > 0$  for every  $\lambda \in (0, 1)$  implies that  $\mu_\lambda$  is an equilibrium strategy for the incumbent, which contradicts the assumption that  $\mu$  is an equilibrium strategy for the incumbent in an equilibrium where there is entry with a positive probability.

$\varepsilon']) > 0$  for any  $\varepsilon' > 0$  implies that the incumbent makes strictly greater profits after playing  $\mu_\lambda$  than after playing  $\mu$ , irrespective of which profit-maximizing strategy the entrant plays. This contradicts the assumption that  $\mu$  is an equilibrium strategy for the incumbent. Therefore Result 1 is true.

**Result 2.** In equilibrium, the measure of the set of consumers who buy from the incumbent at price  $A$  is zero, and the entrant offers (almost) all consumers the price  $A$ .

*Proof of Result 2.* Let us start with the first part. Let us assume that a strictly positive mass of consumers buys at price  $A$  from the incumbent. Notice that this is possible only if the entrant offers some consumers the price  $p$ . Consider now the following deviation for the entrant: replacing all offers of a price  $p$  with the offer of a price  $p - \varepsilon$ . For  $\varepsilon$  small enough, this clearly increases the entrant's profits. Let us prove now the second part of the result. Assume that it is false, i.e. that the entrant offers a price lower than  $A$  with a positive probability. Let  $\nu$  denote the distribution of prices offered by the entrant. The inclusion  $\text{Supp}(\nu) \subset \text{Arg} \max_p \mu([p, \infty))$  implies that the  $A \not\subseteq \text{Arg} \max_p \mu([p, \infty))$ , implying in turn that  $\mu([0, A)) > 0$ . Consider the following price distribution:

$$\begin{aligned}\mu_\lambda(S) &= (1 - \lambda)\mu(S) \text{ if } A \notin S \\ \mu_\lambda(\{A\}) &= \mu(\{A\}) + \lambda\mu([0, A)).\end{aligned}$$

The fact that  $A \in \text{Arg} \max_p \mu([p, \infty))$  (from Result 1) and the construction of  $\mu_\lambda$  imply that for any  $\lambda > 0$ ,  $\text{Arg} \max_p \mu_\lambda([p, \infty)) = \{A\}$ , so that the entrant would choose to offer  $A$  to all consumers if the incumbent played  $\mu_\lambda$ . Therefore, the incumbent's payoff when he plays  $\mu_\lambda$  is at least  $E(p \mathbf{1}_{p < A} | \mu_\lambda)$  while it is at most  $E(p \nu([p, \infty)) \mathbf{1}_{p < A} | \mu)$  if he plays  $\mu$ . If  $\lambda$  is small enough,  $E(p \mathbf{1}_{p < A} | \mu_\lambda) > E(p \nu([p, \infty)) \mathbf{1}_{p < A} | \mu)$ , and  $\mu$  cannot be an optimal strategy for the incumbent, which leads to a contradiction.

**Result 3.** If  $\mu$  is an equilibrium strategy for the incumbent, then there exists  $K \in [0, A)$  such that  $\mu([0, K)) = 0$  and the function  $p \rightarrow p\mu([p, \infty))$  is constant over the interval  $[K, A]$ .

*Proof of Result 3.* Let us define  $M = \max_p (p\mu([p, \infty))) = A\mu([A, \infty))$  (the last equality comes from Result 1), and assume that this result is

not true. This implies that there exists  $A'$  such that  $\mu([A', \infty)) < 1$  and  $A'\mu([A', \infty)) < M$ . We pick such an  $A'$  and some element  $m$  of  $(A'\mu([A', \infty)), M)$ , and we define  $A'' = \inf \{p \mid p'\mu([p', \infty)) < m \text{ for all } p' \in [p, A']\}$ . Obviously  $A'' \leq A'$ . We claim that  $\mu([A'', A']) > 0$ . Two cases are possible. If  $A'' = 0$ , then  $\mu([A'', A']) = \mu([0, A']) = 1 - \mu([A', \infty)) > 0$ . If  $A'' > 0$ , then the inequalities  $A''\mu([A'', \infty)) \geq m$  and  $A''\mu([A', \infty)) \leq A'\mu([A', \infty)) < m$  imply that  $\mu([A'', \infty)) > \mu([A', \infty))$ , or equivalently that  $\mu([A'', A']) > 0$ . Consider now the price distribution  $\mu_\lambda$  given by

$$\begin{aligned}\mu_\lambda(S) &= \mu(S) \text{ if } [A'', A'] \not\subseteq S \\ \mu_\lambda([A'', A']) &= (1 - \lambda)\mu([A'', A']) \\ \mu_\lambda(\{A'\}) &= \mu(\{A'\}) + \lambda\mu([A'', A']).\end{aligned}$$

For all  $p \notin [A'', A']$ ,  $p\mu([p, \infty)) = p\mu_\lambda([p, \infty))$ , and if  $\lambda$  is close enough to zero,  $p\mu_\lambda([p, \infty)) < M$  for all  $p \in [A'', A']$ . Finally, if  $\mu'_\lambda$  is defined by

$$\begin{aligned}\mu_\lambda(S) &= \mu(S) \text{ if } S \text{ does not contain } A \text{ or } A'. \\ \mu'_{\lambda\varepsilon}(\{A'\}) &= (1 - \varepsilon)\mu_\lambda(\{A'\}) \\ \mu'_{\lambda\varepsilon}(\{A\}) &= \mu_\lambda(\{A\}) + \varepsilon\mu_\lambda(\{A'\}),\end{aligned}$$

then  $\arg \max(p\mu'_{\lambda\varepsilon}([p, \infty))) = \{A\}$ . This implies that if an incumbent plays the strategy  $\mu'_{\lambda\varepsilon}$ , the entrant offers a price of  $A$  with probability one, and the incumbent's profit is at least  $E(p.1_{p < A} | \mu'_{\lambda\varepsilon})$ , which converges towards  $E(p.1_{p < A} | \mu_\lambda) > E(p.1_{p < A} | \mu)$  as  $\varepsilon$  converges towards zero, and the right-hand side of the last inequality is the incumbent's payoff in the initial equilibrium (by Result 2). Therefore  $\mu'_{\lambda\varepsilon}$  is a profitable deviation for the incumbent if  $\lambda$  and  $\varepsilon$  are small enough. This proves the result.

**Result 4.** If  $\mu$  is an equilibrium strategy for the incumbent, then it satisfies  $d\mu(p) = \frac{A}{ep^2}$  if  $p \in [0, A]$ ,  $\mu(\{A\}) = \frac{1}{e}$  and  $\mu((A, \infty)) = 0$ .

*Proof of Result 4.* From Result 2, we know that there exist  $K \in [0, A)$  and  $C > 0$  such that  $\mu([0, K)) = 0$ ,  $\mu(\{A\}) = \frac{C}{A}$ , and  $\mu([p, \infty)) = \frac{C}{p}$ , implying  $d\mu(p) = \frac{C}{p^2}$ . The identity  $\mu([K, \infty)) = 1$  implies that  $C = K$ , and the fact that in equilibrium, the entrant offers a price of  $A$  to all consumers, and all consumers offered  $A$  by both firms buy from the entrant implies that the incumbent's revenue is equal to

$$\int_K^A \frac{K}{p} dp = K [\log A - \log K],$$

which is maximized at

$$K = \frac{A}{e},$$

leading to the result. The incumbent's revenue is then equal to  $K[\log A - \log K] = \frac{A}{e}\log(e) = \frac{A}{e}$ , and the entrant's revenue is equal to  $A\mu(\{A\}) = \frac{A}{e}$ .

**PART 3.** From Part I we know that the maximal revenue that the incumbent can earn by playing a strategy leading to no entry in the second stage of the game is the one yielded by the strategy  $\mu^*$ , that is,  $B(1 + \log(\frac{A}{B}))$ , and the corresponding profit is  $B\log(\frac{A}{B})$ . From Part II we know that the maximal revenue that the incumbent can earn by playing a strategy leading to entry in the second stage of the game is the one yielded by the strategy  $\hat{\mu}$ , that is,  $\frac{A}{e} - B$ . This implies that in equilibrium the incumbent chooses to play  $\mu^*$  (resp.  $\hat{\mu}$ ) if  $B\log(\frac{A}{B})$  is strictly greater (resp. strictly smaller) than  $\frac{A}{e} - B$ , or in other words if  $\frac{A}{B}$  is strictly greater (resp. strictly smaller) than  $\eta$ , defined as the solution to the equation  $x = e(\log(x) + 1)$ .

#### PROOF OF PROPOSITION 4

It is enough to prove the result in the case  $B = 0$ . Let  $\mu_1, \mu_2$  denote the equilibrium price distributions chosen by firms 1 and 2 respectively, and let  $\mu$  denote the distribution of  $\text{Min}(p_1, p_2)$ , where  $p_1, p_2$  are drawn respectively and independently from  $\mu_1, \mu_2$ . For  $i = 1, 2$ , let us define  $B_i = \text{Max}\{p \mid \mu_i(0, p) < 1\}$ . We first show that  $\text{Min}(B_1, B_2) = 0$ . Let us assume that this is not true. First, equilibrium implies that  $B_1 = B_2$  : since  $B_2 > 0$ , any price offer by firm 1 in the interval  $(0, B_2)$  is accepted with a positive probability and yields a positive profit, while price offers strictly above  $B_2$  are never picked. This argument implies that  $B_1 = \text{Max}\{p \mid \mu_1(0, p) < 1\} \leq B_2$ . The same argument implies that  $B_2 \leq B_1$ , so that  $B_1 = B_2 > 0$ . Let  $B'$  denote the common value of  $B_1$  and  $B_2$ . We show now that this leads to a contradiction. Assume first that  $\mu_1(\{B'\}) = \mu_2(\{B'\}) = 0$ . This implies that for any  $i = 1, 2$ ,  $\lim_{\varepsilon \rightarrow 0} (B' - \varepsilon) \mu_i(B' - \varepsilon, \infty) = 0$ , so that for any  $B'' > 0$ ,

$$\lim_{\varepsilon \rightarrow 0} \frac{B'' \mu(B'', B')}{(B' - \varepsilon) \mu(B' - \varepsilon, B')},$$

implying that if  $\varepsilon$  is small enough, no optimal strategy involves price offers in  $(B' - \varepsilon, B']$ , contradicting the definition of  $B'$ . Assume now that  $\mu_1(\{B'\}) > 0$  and  $\mu_2(\{B'\}) = 0$ . Then, when it offers the price  $B'$ , firm 1 makes a zero expected profit because almost all consumers are offered lower prices by firm 2, while offering a price  $B'' \in (B, B')$  would yield firm 1 a strictly positive profit, because  $\mu_2(B'', \infty) > 0$ . This means that  $\mu_1$  is not a best response to  $\mu_2$ , which is impossible. Finally, assume that  $\mu_1(\{B'\}) > 0$  and  $\mu_2(\{B'\}) > 0$ , and (for example) that consumers offered  $B'$  by both firms pick firm 2 with a probability  $\alpha > 0$ . Then, offering  $B'$  yields on average a profit equal to  $\mu_2(\{B'\})(1 - \alpha)B'$ , while offering  $B' - \varepsilon$  yields on average a profit greater than  $\mu_2(\{B'\})(B' - \varepsilon)$ , which is greater than  $\mu_2(\{B'\})(1 - \alpha)B'$  if  $\varepsilon$  is small. This implies that offering  $B'$  cannot be part of a best response price distribution for firm 1, leading to a contradiction.

We proved therefore that  $\text{Min}(B_1, B_2) = 0$ . Assume for example that  $B_2 = 0$ . We show that  $B_1 = 0$ . Assume it is not true. Then, firm 2, by offering the price  $\frac{B_1}{2}$ , would earn in expectation at least  $\frac{B_1}{2}\mu_1\left((\frac{B_1}{2}, B_2]\right) > 0$ , which is greater than its equilibrium payoff of zero, leading to a contradiction. Therefore  $\text{Max}(B_1, B_2) = 0$ . Since it is never profitable to offer negative prices, this proves that in equilibrium both firms charge zero and make zero profits.

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